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Supersymmetric tachyons

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Abstract. In this paper we discuss the supersymmetric tachyon and its applications. Both unitary and non-unitary representations for the superalgebra are examined. If we abandon the standpoint that any elementary particle in relativistic quantum theory must be described by unitary irreducible representations of the Poincaré algebra or the superalgebra, then we can construct the supersymmetric invariant action for supersymmetric tachyons. The scalar neutrino's mass is lighter than the photino's mass if the neutrino is the tachyon and the photon is a massless particle in the simplest supersymmetry-breaking model. There is a possibility that the cold dark matter consists of scalar neutrinos.

1. Introduction

The idea that tachyons might exist has attracted some attention in the literature over the past two decades (cf, e.g., Recami 1978). However, most work to date (Arons and Suadshan 1968, Dhar and Suadshan 1968, Feinberg 1967, 1978) has been concerned with scalar tachyons. Chodos *et al* (1985) suggested that at least one of the known neutrinos might possibly be a fermionic tachyon and examined the available data on the neutrino mass from pion decay $\pi^+ \to \mu^+ \nu$ from this viewpoint. van Dam *et al* (1985) pointed out that this suggestion cannot be realised in the framework of field theory. However, their conclusion is based on the unitarity restriction: any elementary particle in relativistic quantum theory must be described by unitary irreducible representations of Poincaré algebra or its supersymmetric generalisation.

In this paper we discuss the supersymmetric tachyon and its applications and examine both unitary and non-unitary representations of the $N = 1$ superalgebra. We extend Wigner's work (1963) on the wave equation to the supersymmetry case by using a formalism developed by Dirac (1963) and others. If we accept the standpoint of the unitary restriction, then we can conclude that even though a chiral or a gauge super-multipet can be written easily for imaginary mass particles, such a supermultiplet does not describe the tachyonic one. On the other hand, if we abandon the unitary restriction, then we can construct a supersymmetric action for supersymmetric tachyons. A realistic particle spectrum demands the breaking of supersymmetry. It is reasonable that the mass of the scalar neutrino is less than that of the photino if the neutrino is the tachyon and the photon is a massless particle. We explore the possibility that the scalar neutrino is the lightest supersymmetry partner. This assumption implies $m_{\tilde{\nu}} \leqslant 2$ GeV if $\rho_{\tilde{\nu},0} \sim 3H/8\pi G$.

2. Unitary representation

There are fourteen generators in the $N=1$ superalgebra, namely, four generators of the translation P_μ, six generators of the Lorentz translation $M_{\mu\nu}$ and four spinoral generators Q_a, which satisfy the Majorana condition $Q_a = C_{ab}(\bar{Q}^{\mathrm{T}})_b$. The algebra is given by

$$
\begin{aligned}
&[P_\mu, P_\nu] = 0 \\
&[M_{\mu\nu}, P_\lambda] = -\mathrm{i}(g_{\mu\lambda}P_\nu - g_{\nu\lambda}P_\mu) \\
&[M_{\mu\nu}, M_{\lambda\rho}] = -\mathrm{i}(g_{\mu\lambda}M_{\nu\rho} + g_{\nu\rho}M_{\mu\lambda} - g_{\nu\lambda}M_{\mu\rho} - g_{\mu\rho}M_{\nu\lambda}) \\
&[P_\mu, Q_a] = 0 \\
&[M_{\mu\nu}, Q_a] = -(\sigma_{\mu\nu})_{ab}Q_b \\
&\{Q_a, Q_b\} = (\gamma_\mu C)_{ab}P^\mu
\end{aligned}
\tag{1}
$$

where $\sigma_{\mu\nu} = \frac{1}{4}\mathrm{i}[\gamma_\mu, \gamma_\nu]$, $C = \mathrm{i}\gamma^2\gamma^0$ and γ_μ are the usual Dirac matrices which are taken as follows:

$$
\gamma^0 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \qquad \gamma^i = \begin{bmatrix} 0 & -\sigma^i \\ \sigma^i & 0 \end{bmatrix} \qquad \gamma^5 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}. \tag{2}
$$

The Casimir operators of the superalgebra are P^2 and $C_{\mu\nu}C^{\mu\nu}$:

$$
C_{\mu\nu} = (W_\mu + \tfrac{1}{4}\bar{Q}\gamma_\mu\gamma^5 Q)P_\nu - (W_\nu + \tfrac{1}{4}\bar{Q}\gamma_\nu\gamma^5 Q)P_\mu \tag{3}
$$

where W_μ is the Pauli–Lubanski vector. The irreducible representations may be labelled by the eigenvalues of these Casimir operators.

The Majorana condition is

$$
(Q_1, Q_2, Q_3, Q_4)^{\mathrm{T}} = (Q_1, Q_2, Q_2^*, -Q_1^*)^{+}. \tag{4}
$$

Consider any irreducible representation $|\beta\rangle$ of the superalgebra. There will be a state $|\alpha\rangle = Q_1Q_2|\beta\rangle$ satisfying $Q_1|\alpha\rangle = Q_2|\alpha\rangle = 0$; namely, $|\alpha\rangle$ is a Clifford vacuum.

Next, we consider a Hilbert space H of complex-valued functions $\alpha(p, \xi_1, \xi_2)$ for the Clifford vacuum, where ξ_1 and ξ_2 are harmonic oscillator coordinates. In this Hilbert space, a unitary representation is supported via

$$
\begin{aligned}
&P^\mu\alpha(p, \xi_1, \xi_2) = p^\mu\alpha(p, \xi_1, \xi_2) \\
&M^{\mu\nu}\alpha(p, \xi_1, \xi_2) = \left(s^{\mu\nu} - \mathrm{i}p^\mu\frac{\partial}{\partial p_\nu} + \mathrm{i}\,p^\nu\frac{\partial}{\partial p_\mu}\right)\alpha(p, \xi_1, \xi_2)
\end{aligned}
\tag{5}
$$

where

$$
s^{\mu\nu} = \frac{1}{4}\begin{pmatrix}
0 & \xi_1^2 - \pi_1^2 - \xi_2^2 + \pi_2^2 & 2\pi_1\pi_2 - 2\xi_1\xi_2 & 2\xi_1\pi_1 + 2\pi_2\xi_2 \\
\pi^2 - \xi^2 + \pi_2^2 + \xi^2 & 0 & 2\xi_2\pi_1 - 2\xi_1\pi_2 & 2\xi_1\xi_2 + 2\pi_1\pi_2 \\
2\xi_1\xi_2 - 2\pi_1\pi_2 & 2\xi_1\pi_2 - 2\xi_2\pi_1 & 0 & \xi_1^2 + \pi_1^2 - \xi_2^2 - \pi_2^2 \\
-2\xi_1\pi_1 - 2\pi_2\xi_2 & -2\xi_1\xi_2 - 2\pi_1\pi_2 & \xi_2^2 + \pi_2^2 - \xi_1^2 - \pi_1^2 & 0
\end{pmatrix}.
$$

This representation may be reduced by imposing covariant constraints:

$$
\begin{aligned}
&(P^\mu P_\mu - m^2)\alpha(p, \xi_1, \xi_2) = 0 \\
&(P^\mu V_\mu - \omega)\alpha(p, \xi_1, \xi_2) = 0
\end{aligned}
\tag{6}
$$

where

$$V_0=\tfrac{1}{4}(\xi_1^2+\pi_1^2+\xi_1^2+\pi_2^2) \qquad V_1=\tfrac{1}{2}(\xi_2\pi_2-\xi_1\pi_1)$$
$$V_2=\tfrac{1}{2}(\xi_1\pi_2+\xi_2\pi_1) \qquad V_3=\tfrac{1}{4}(\xi_1^2-\pi_1^2+\xi_2^2-\pi_2^2) \tag{7}$$

and π_k is the conjugated momentum of ξ_k,

$$[\xi_j, \xi_k]=[\pi_j, \pi_k]=0 \qquad [\xi_j, \pi_k]=\mathrm{i}\delta_{jk} \qquad j, k=1, 2. \tag{8}$$

In the case of $m^2<0$, we may take $p^\mu=(0, 0, 0, m)$ as the frame and $P^\mu V_\mu$ is reduced to

$$P^\mu V_\mu=\tfrac{1}{4}|m|(\pi_1^2+\pi_2^2-\xi_1^2-\xi_2^2). \tag{9}$$

The little group is spanned by S_{12}, S_{10} and S_{20}. Its generators commute with V_3 which may be used to reduce this representation. From (9) and (6), we find that ω can have any real eigenvalue and it is related to the ω vector by

$$\omega^\mu\omega_\mu=\tfrac{1}{4}|m|^2-\omega^2. \tag{10}$$

For a fixed ω there is a further reduction which can be seen easily after diagonalising S_{12}, the helicity operator for the frame $p^\mu=(0, 0, 0, |m|)$. The other generators of the little group, S_{10} and S_{20}, raise or lower this helicity by units of ± 1. Hence for any fixed ω, there are two irreducible representations under Poincaré algebra: one contains all the half-integer helicity states and the other contains all the integer ones. The operator ω is Hermitian, so the eigenvalues of ω^2 are non-negative. Since $[S_{12}, Q_2^*]=-\tfrac{1}{2}Q_2^*$ and $[S_{12}, Q_1^*]=\tfrac{1}{2}Q_1^*$, we have irreducible representations of the superalgebra $(|\alpha\rangle,\ Q_1^*|\alpha\rangle, Q_2^*|\alpha\rangle, Q_1^*Q_2^*|\alpha\rangle)$, which carry a fourfold infinite number of linearly independent helicity states for a given 4-momentum.

In the case of $m^2=0$, we may taken $p^\mu=(E, 0, 0, E)$ as a typical frame and have

$$P^\mu V_\mu=E(V_0-V_3)=\tfrac{1}{2}E(\pi_1^2+\pi_2^2). \tag{11}$$

The little group is spanned by S_{12} and two 'translations', S_{10}-S_{13} and S_{20}-S_{23}. These two 'translations' commute with V_0-V_3 which may be used to label the irreducible representations. Substituting (11) in (6), we obtain

$$\omega=\tfrac{1}{2}E(\pi_1^2+\pi_2^2)\equiv\Xi. \tag{12}$$

Ξ can be any non-negative.

For a fixed Ξ there is a further reduction of the representation. We consider that the helicity operator S_{12} for $p^\mu=(E, 0, 0, E)$ does not change the quantum number of the degenerate oscillators. The operators S_{10}-S_{13} and S_{10}-S_{23} change the helicity by $+1$. Thus, for a given Ξ there are two irreducible representations under the Poincaré superalgebra: one contains all the half-integer states and the other contains all the integer ones. Since $[S_{12}, Q_1^*]=\tfrac{1}{2}Q_1^*$, we have another reducible representation of the superalgebra $(Q_1^*|\alpha\rangle, |\alpha\rangle)$ which carries a twofold infinite number of linearly independent helicity states. For $\Xi=0$ there is a further decoupling so that the representation is reduced to the usual supersymmetry multiplet with spins $(j, j-\tfrac{1}{2})$.

In $N=1$ supersymmetry theory, the familiar supermultiplets include a chiral doublet $(\tfrac{1}{2}, 0)$, a gauge doublet $(1, \tfrac{1}{2})$ and a graviton-gravitino doublet $(2, \tfrac{3}{2})$. However, the $\Xi=0$ massless irreducible representations which correspond to tachyonic supermultiplets carry an infinite number of linearly independent helicity states for any given 4-momentum vector p^μ. Thus they do not fit any familiar supermultiplet having a finite number of independent components for a fixed p_μ.

Many theoretical works are based on the simple restriction that any elementary particle in relativistic quantum theory must be described by a unitary representation of the Poincaré algebra or superalgebra. If we accept this standpoint, then we may conclude that, although a chiral or a gauge supermultiplet can be written easily for an imaginary mass, such a supermultiplet does not describe tachyons.

3. Non-unitary representation

The three-parameter Lie algebras have, as their complex extensions, isomorphic simple complex algebras $A_1 \sim B_1 \sim C_1$. The real forms of these algebras may be divided into three compact isomorphic Lie algebras so(3) ~ su(2) ~ sp(2) and four non-compact isomorphic Lie algebras so(2, 1) ~ so(1, 1) ~ sl(2, R) ~ sp(2, R). In addition, we can consider a three-parameter Euclidean algebra in two dimensions E_2, comprised of the semidirect sum $E_2 \sim T_2 \oplus \mathrm{so}(2)$ where T_2 is the translation in the two dimensions.

Barut (1967) has shown that the construction of the representations of the eight algebras mentioned above may be given a unified treatment. If the generators of the Lie algebras are written as

$$S_{ij} = -S_{ji} \qquad i, j = 1, 2, 3 \tag{13}$$

and we define

$$S_{\pm} = (S_{13} + \mathrm{i}S_{23})/\sqrt{2} \tag{14}$$

then the commutation relations become

$$[S_+, S_-] = g_{33}S_{12} \qquad [S_{12}, S_{\pm}] = +S_{\pm} \tag{15}$$

where $g_{33} = 1$ for so(3) ~ su(2) ~ sp(2), $g_{33} = -1$ for so(2, 1) ~ su(1, 1) ~ sl(2, R) ~ sp(2, R) and $g_{33} = 0$ for E_2. For any unitary representation we must have $S_{ij}^{+} = S_{ij}$ and $S_+^{\dagger} = S_-$. If we put

$$S_{12} = \tfrac{1}{2}\sigma_3 \qquad S_{\pm} = \tfrac{1}{4}\sqrt{2g_{33}}(\sigma_1 + \mathrm{i}\sigma_2) \tag{16}$$

then we obtain the elementary representations for the three classes of Lie algebra. For $g_{33} = 1$ we obtain the elementary representation for so(3) ~ su(2) ~ sp(2), which is unitary and irreducible, while for the $g_{33} = -1$ we get the elementary representations for so(2, 1) ~ su(1, 1) ~ sl(2, R) ~ sp(2, R), which are irreducible but non-unitary. The highest weight of this spinor representation is $\frac{1}{2}$. The eigenvalue of the Casimir invariant $C^2 = g_{33}S_{12}(S_{12}+1) + 2S_-S_+$ is readily found to be

$$C^2 = \tfrac{3}{4}g_{33}. \tag{17}$$

Next we show that the non-unitary representation of the superalgebra accords with the method mentioned above.

Haag *et al* (1975) proved that the superalgebra (and its extensions with central charges) is the only graded Lie algebra of symmetries of the S matrix which is consistent with relativistic quantum field theory. The extended superalgebra can be written as

$$\begin{aligned} &\{Q_\alpha{}^A, \bar{Q}_{\dot\beta B}\} = 2\sigma^m_{\alpha\dot\beta} P_m \delta^A{}_B \\ &\{Q_\alpha{}^A, Q_\beta{}^B\} = \{\bar{Q}_{\dot\alpha A}, \bar{Q}_{\dot\beta B}\} = 0 \\ &[P_m, Q_\alpha{}^A] = [P_m, \bar{Q}_{\dot\alpha A}] = 0 \\ &[P_m, P_n] = 0 \end{aligned} \tag{18}$$

where the indices $(\alpha, \beta, \ldots, \dot{\alpha}, \dot{\beta}, \ldots)$ run from 1 to 2 and denote two-component Weyl spinors. The indices $(m, n, \ldots)$ identify Lorentz indices. The indices $(A, B, \ldots)$ refer to an internal space and run from 1 to N. The algebra with $N=1$ is called the superalgebra, while those with $N>1$ are called extended superalgebras.

In the case of $m^2<0$, we have taken $p^\mu=(0,0,0,|m|)$ as a typical frame, and the superalgebra reduces to

$$\{Q_\alpha{}^A, \bar{Q}_{\dot{\beta}B}\} = 2|m|(-1)^\alpha \delta_{\alpha\dot{\beta}} \delta^A{}_B. \tag{19}$$

The generators Q may be rescaled

$$\begin{aligned} b_\alpha{}^A &= ((-1)^\alpha/2|m|)^{1/2} Q_\alpha{}^A \\ (b_\alpha{}^A)^\dagger &= ((-1)^\alpha/2|m|)^{1/2} \bar{Q}_{\dot{\alpha}A} \end{aligned} \tag{20}$$

so as to show that (19) is the algebra of $2N$ fermionic creation and annihilation operators, $(b_\alpha{}^A)^\dagger$ and $b_\alpha{}^A$,

$$\begin{aligned} \{b_\alpha{}^A, (b_\beta{}^B)^\dagger\} &= \delta_{\alpha\beta}\delta^A{}_\beta \\ \{b_\alpha{}^A, b_\beta{}^B\} &= \{(b_\alpha{}^A)^\dagger, (b_\beta{}^B)^\dagger\} = 0. \end{aligned} \tag{21}$$

The representations of this algebra are well known. They are constructed from a Clifford vacuum $|\alpha\rangle$. The Clifford vacuum is defined through the condition

$$b_\alpha{}^A|\alpha\rangle = 0. \tag{22}$$

The states are built by applying the creation operators $(b_\alpha{}^A)^\dagger$ to $|\alpha\rangle$

$$|\alpha^{(n)}{}_{A_1}^{\alpha_1} \ldots {}_{A_n}^{\alpha_n}\rangle = (n!)^{-1/2} (b_{\alpha_1}{}^{A_1}) \ldots (b_{\alpha_n}{}^{A_n})|\alpha\rangle. \tag{23}$$

Because of the anticommuteness of $(b_\alpha{}^A)^\dagger$, $|\alpha^n\rangle$ is antisymmetric under the exchange of two pairs of indices $\alpha_i A_i$, $\alpha_j A_j$. Each pair of indices takes $2N$ different values, so n must be less than or equal to $2N$. For any given n, there are $\binom{2N}{n}$ different states.

In the Weyl basis of Dirac matrices, the Majorana spinors contain only one Weyl spinor

$$Q_M^A = \begin{pmatrix} Q_\alpha{}^A \\ \bar{Q}^{\dot{\alpha}A} \end{pmatrix}. \tag{24}$$

For the superalgebra there are two Casimir operators, P^2 and C^2, where

$$\begin{aligned} C^2 &= C_{\mu\nu} C^{\mu\nu} \\ C_{\mu\nu} &= (\tfrac{1}{2}\varepsilon_{\mu\nu\lambda\rho} M^{\nu\lambda} P^\rho + \tfrac{1}{4}\bar{Q}_{MA}\gamma_\mu\gamma^5 Q_M^A) P_\nu. \end{aligned} \tag{25}$$

The irreducible represnetations may be labelled by the eigenvalues of the Casimir operators, so we seek these eigenvalues of C^2. In a typical frame $P^\mu = (0,0,0,m)$, C^2 becomes

$$\begin{aligned} C^2 &= 2|m|^2 \omega_k \omega^k \\ \omega_k &= \tfrac{1}{2}\varepsilon_{k\nu\lambda\rho} M^{\nu\lambda} P^\rho + \tfrac{1}{4}\bar{Q}_{MA}\gamma_k\gamma^5 Q_M^A \qquad k=0,1,2 \end{aligned} \tag{26}$$

where ω_k is called the ω spin and satisfies the following commutation relations:

$$\begin{aligned} &[\omega_0, \omega_1] = \mathrm{i}\omega_2 \qquad [\omega_1, \omega_2] = -\mathrm{i}\omega_0 \qquad [\omega_2, \omega_0] = \mathrm{i}\omega_1 \\ &[\omega_k, Q_\alpha^A] = [\omega_k, \bar{Q}^{\dot{\alpha}A}] = 0. \end{aligned} \tag{27}$$

Now $\omega^k\omega_k$ is a Casimir operator and ω_k satisfies the commutation relation of su(1, 1). The eigenvalues of ω^2 will therefore have the form $-j(j+1)$, where j is integer or half-integer. The irreducible non-unitary representations of the superalgebra may then be labelled by the eigenvalues of P^2 and ω^2.

When the Clifford vacuum $|\alpha_j\rangle$ has spin j ($j>0$), it belongs to a $(2j+1)$-dimensional non-unitary representation of the little group su(1, 1). So the dimension of the representation of the superalgebra is $d=2^{2N}(2j+1)$.

The results are summarised in table 1 for the cases of $N=1$, 2, 3 and 4.

In fact, there does not as yet exist a completely satisfactory quantum field theory for any type of tachyons (Kamoi and Kamefuchi 1971). Chodos *et al* (1985) suggested that such difficulties cannot be used to exclude *a priori* the existence of tachyons and more theoretical work is required to determine physically acceptable modifications of the usual non-tachyonic quantum field theory.

If we abandon the unitary principle, then we can construct a tachyonic supersymmetry action.

Table 1. $N\leq 4$ non-unitary tachyonic representation.

		Spin				
N	Vacuum	0	$\frac{1}{2}$	1	$\frac{3}{2}$	2
1	$\|\alpha_0\rangle$	2	1	0	0	0
	$\|\alpha_{1/2}\rangle$	1	2	1	0	0
	$\|\alpha_1\rangle$	0	1	2	1	0
	$\|\alpha_{3/2}\rangle$	0	0	0	2	1
2	$\|\alpha_0\rangle$	5	4	1	0	0
	$\|\alpha_{1/2}\rangle$	4	6	4	1	0
	$\|\alpha_1\rangle$	1	4	6	4	1
3	$\|\alpha_0\rangle$	14	14	6	1	0
	$\|\alpha_{1/2}\rangle$	14	20	15	6	1
4	$\|\alpha_0\rangle$	42	48	27	8	1

4. Supersymmetric tachyons and their applications

It is convenient to describe the fermions in the chiral supermultiplets by using the left-handed Weyl field. A Dirac particle then gets counted twice; once for its left-handed particle state and once for its left-handed antiparticle state. Let anticommuting parameters ξ^α, $\bar{\xi}_{\dot{\alpha}}$ satisfy

$$\{\xi^\alpha, \xi^\beta\}=\{\xi^\alpha, Q_\beta\}=\ldots=[P_m, \xi^\alpha]=0. \tag{28}$$

The component multiplet with supersymmetry transformation is as follows:

$$\begin{aligned}
&\delta_\xi A=\sqrt{2}\xi\psi \\
&\delta_\xi\psi=\mathrm{i}\sqrt{2}\sigma^m\bar{\xi}\partial_m A+\sqrt{2}\xi F \\
&\delta_\xi F=\mathrm{i}\sqrt{2}\bar{\xi}\bar{\sigma}^m\partial_m\psi.
\end{aligned} \tag{29}$$

These fields can form a linear representation of the $N=1$ superalgebra. If A has dimension 1, then ψ has dimension $\frac{3}{2}$ while F, which is the auxiliary field, has dimension 2. The invariant actions are given by

$$\mathscr{L} = \mathrm{i}\partial_n\bar{\psi}\bar{\sigma}^n\psi + A^*\Box A + F^*F - m(\tfrac{1}{2}\bar{\psi}\bar{\psi} - \tfrac{1}{2}\psi\psi + AF - A^*F^*) \tag{30}$$

or

$$\mathscr{L}' = \mathrm{i}\partial_n\bar{\psi}\bar{\sigma}^n\psi + A^*\Box A + F^*F - m(\tfrac{1}{2}\psi\psi + \tfrac{1}{2}\bar{\psi}\bar{\psi} - AF - A^*F^*). \tag{31}$$

There is an important difference between $\mathscr{L}$ and $\mathscr{L}'$. Equation (30) describes a normal chiral supermultiplet, while (31) describes a tachyonic one. Because of the little group for spacelike momenta is non-compact and its unitary representations are therefore infinite dimensional, the Fock space of the theory given by (31) involves a non-unitary representation.

If the neutrino is a spinorial tachyon (Chodos *et al* 1985) then its supersymmetric partner, the scalar neutrino, will also be a tachyon since unbroken supersymmetry implies mass degeneracy between bosons and fermions belonging to the same supermultiplet. The scalar neutrino can be a normal particle if the supersymmetry breaking has occurred. The most elegant and plausible mechanism for supersymmetry breaking is a spontaneous-breaking one, as in the case of gauge theories. Spontaneous global supersymmetry breaking requires a vacuum that is not supersymmetric. In this case, there must be some state $\tilde{G}$ (called the goldstino). It couples to the vacuum via a supersymmetry charge Q whose element is

$$\langle 0|Q|\tilde{G}\rangle \equiv m_s^2 \neq 0 \tag{32}$$

where the parameter m characterises the scale of supersymmetry breaking. From the superalgebra and translation invariance of the vacuum we infer

$$\langle 0|P_0|0\rangle = m_s^4 = 0. \tag{33}$$

There will be global supersymmetry breaking if, and only if, the vacuum energy does not vanish.

The elements of superspace are denoted as $z=(x, \theta, \bar{\theta})$. The chiral superfields are defined

$$\begin{aligned}\Phi_i = {} & A_i(x) + \mathrm{i}\theta\sigma^m\bar{\theta}\partial_m A(x) + \tfrac{1}{4}\theta\theta\bar{\theta}\bar{\theta}\Box A_i(x) \\ & + \sqrt{2}\theta\psi_i(x) - (\mathrm{i}/\sqrt{2})\theta\theta\partial_m\psi_i(x)\sigma^m\bar{\theta} + \theta\theta F(x).\end{aligned} \tag{34}$$

The simplest model (Farrar and Fayet 1978a, b) has three gauge-singlet chiral superfields ϕ_1, ϕ_2, ϕ_3 and the superpotential is

$$P = \lambda_1\phi_1^2\phi_2 + \lambda_2(\phi_1^2 - \mu)\phi_3. \tag{35}$$

If we are restricted in a weak-coupling model, then the following 'low-energy theorem' holds

$$\Delta m^2 = g_i M_s^2 \tag{36}$$

where g_i is the coupling constant of the ith multiplet of the goldstino. The mass of the scalar neutrino and the photino are $m_{\tilde{\nu}}$ and $m_{\tilde{\gamma}}$, respectively, and are given by

$$\begin{aligned} m_{\tilde{\nu}}^2 &= g_i M_s^2 - |m_\nu^2| \\ m_{\tilde{\gamma}}^2 &= g_2 M_s^2. \end{aligned} \tag{37}$$

The mass of the scalar neutrino is lighter than that of the photino, from (37), if neutrinos are tachyons and the photon is a massless particle in the simplest model ($g_1 = g_2$). The lightest supersymmetric partner particle is stable because of R parity (Farrar and Fayet 1978a, b, Farrar and Weinberg 1983), a reflection symmetry equivalent to

$$R = (-1)^{3(B-L)}(-1)^F \tag{38}$$

where B, L and F are baryon, lepton and fermion numbers respectively. R parity is therefore an exact symmetry in any theory in which $(B-L)$ and F are conserved. In other supersymmetric theories, the lightest supersymmetric partner is almost the photino of mass $m_{\tilde{\gamma}} \geq \frac{1}{2}$ GeV. This lower limit corresponds to the cosmological critical density and is dependent on the theoretical parameters controlling the photino's mass and its interactions (Goldberg 1983). In the scalar neutrino case, the difference from the photino as the lightest supersymmetric partner is that the scalar neutrino annihilation rate can be much larger (Barnett *et al* 1983), so there is no lower limit on the scalar neutrino mass from the cosmological density. However, the assumption that the scalar neutrino is the lightest supersymmetry partner implies that $m_{\tilde{\nu}} \leq 2$ GeV if $\rho_{\tilde{\nu},0} \sim 3H_0^2/8\pi G$. If we require $\rho_{\tilde{\nu}}^0 \sim \rho_c^0$, then $m_{\tilde{\nu}} \leq 2$ GeV; or if $\rho_{\tilde{\nu}}^0 \sim 0.1\rho_c^0$, then $m_{\tilde{\nu}} \leq 1$ GeV.

There is a possibility that the cold dark matter ($M_D < 10^8 M_\odot$) consists of scalar neutrinos. It has been shown (Blumenthal *et al* 1984) that good agreement with the galaxy and cluster data is obtained in the cold dark matter model for a Zeldovich spectrum of primordial fluctuations. The model also appears to be reasonably consistent with the observed large-scale clustering, including superclusters and voids.

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